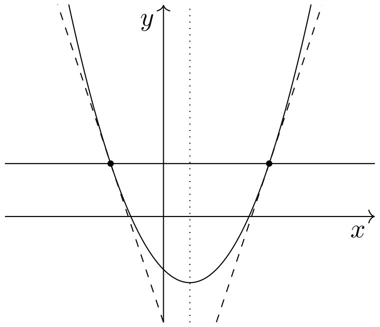
1701. We rearrange as follows:

$$\begin{aligned} & x^2\frac{dy}{dx} - 2\sin x\cos y = \sin x \\ \implies & x^2\frac{dy}{dx} = \sin x(1+2\cos y) \\ \implies & \frac{1}{1+2\cos y}\frac{dy}{dx} = \frac{\sin x}{x^2}. \end{aligned}$$

1702. Consider the graph $y = \mathrm{g}(x)$. This has a line of symmetry of the form $x = k$, through its vertex.



Since $\mathrm{g}(a) = \mathrm{g}(b)$, the points $x = a$ and $x = b$ must be symmetrical in this line. So, the tangents to $y = \mathrm{g}(x)$ at these two points are also reflections of one another. Hence, $\mathrm{g}'(a) = -\mathrm{g}'(b)$. This gives $\mathrm{g}'(a) + \mathrm{g}'(b) = 0$. □

1703. (a) Using log rules,

$$\begin{aligned} & \ln y = 2\ln x - \ln\sqrt{x} \\ \implies & \ln y = \ln\frac{x^2}{\sqrt{x}} \\ \implies & y = x^{\frac{3}{2}}. \end{aligned}$$

(b) Using the fact that $\log_a b \equiv \log_{\sqrt{a}}\sqrt{b}$,

$$\begin{aligned} & \log_2 y = 3\log_2 x + \log_4 x \\ \implies & \log_2 y = \log_2 x^3 + \log_2\sqrt{x} \\ \implies & \log_2 y = \log_2 x^{\frac{7}{2}} \\ \implies & y = x^{\frac{7}{2}}. \end{aligned}$$

1704. This is a correct description of the driving force on the bicycle. Stepping on the pedals is an attempt to make the part of the wheel in contact with the road travel backwards. Friction resists this. Equal and opposite frictional forces act backwards on the road and forwards on the bicycle wheel.

1705. (a) These are APs: $a_n = 2n - 1$ and $b_n = 2n$.

(b) $c_n = a_n b_n$.

(c) Combining the first two parts of the question, $c_n = 2n(2n-1)$. So, we require

$$\begin{aligned} & 2n(2n-1) = 1000 \\ \implies & 4n^2 - 2n - 1000 = 0 \\ \implies & n \approx -15.6, 16.1. \end{aligned}$$

The first term to exceed 1000 is $c_{17} = 1122$.

1706. A fraction can be zero only when its numerator is zero. The first factor is a difference of two squares:

$$\begin{aligned} & (x^4 - a^4)(x^4 + b^4) = 0 \\ \implies & (x^2 - a^2)(x^2 + a^2)(x^4 + b^4) = 0 \\ \implies & (x - a)(x + a)(x^2 + a^2)(x^4 + b^4) = 0. \end{aligned}$$

Since $a, b \neq 0$, only the first two factors have roots $x = \pm a$. Neither makes the denominator zero, as a^4 and c^4 are distinct. So, $x \in \{a, -a\}$.

1707. (a) Tangent and radius are perpendicular, so θ is in a right-angled triangle. The radius is 1, so the longer part of the side has length $\tan\theta$. Then, by symmetry, the shorter part has length $\cot\theta$, because we have switched θ for $90° - \theta$, which switches the values of sin and cos.

(b) Using the value of the perimeter and part (a), $\tan\theta + \cot\theta = \frac{16}{4} = 4$. Multiplying by $\tan\theta$ and rearranging gives $\tan^2\theta - 4\tan\theta + 1 = 0$, as required.

(c) This is a quadratic in $\tan\theta$. Solving with the formula, $\tan\theta = 2 + \sqrt{3}$, which gives $\theta = 75°$. So, the interior angles are 150° and 30°.

— Nota Bene —

Taking the negative root $\tan\theta = 2 - \sqrt{3}$ gives $\theta = 15°$. The problem is symmetrical if sin and cos are switched, so this produces the same set of interior angles.

1708. Consider the function $\mathrm{g}(x) = \sin(2\pi x)$. Since the sine function has period 2π, g has period 1. In other words, $\mathrm{g}(x+1) = \mathrm{g}(x)$ for all $x \in R$. So,

$$\begin{aligned} \mathrm{f}(x+1) &= \mathrm{g}(x+1) + (x+1) \\ &= \mathrm{g}(x) + x + 1 \\ &> \mathrm{g}(x) + x \\ &= \mathrm{f}(x). \end{aligned}$$

So, f is a counterexample to the claim.

— Nota Bene —

The key point here is that *increasing* is a technical term whose sense (like most things in calculus) is local/instantaneous rather than global/average. If $\mathrm{f}(x+1) > \mathrm{f}(x)$ for all x, then $\mathrm{f}(x)$ does "get bigger" in a general sense, but that doesn't mean that f is increasing in a formal sense.

1709. (a) Splitting the integral up,

$$\begin{aligned} & \int_0^1 1 + \mathrm{f}(x)\,dx \\ &= \Big[x\Big]_0^1 + \int_0^1 \mathrm{f}(x)\,dx \\ &= (1) - (0) + 2 \\ &= 3. \end{aligned}$$

(b) Using the same technique,

$$\begin{aligned}&\int_0^1 3\big(x^2+\mathrm{f}(x)\big)\,dx\\ &=\Big[x^3\Big]_0^1+3\int_0^1 \mathrm{f}(x)\,dx\\ &=(1)-(0)+3\times 2\\ &=7.\end{aligned}$$

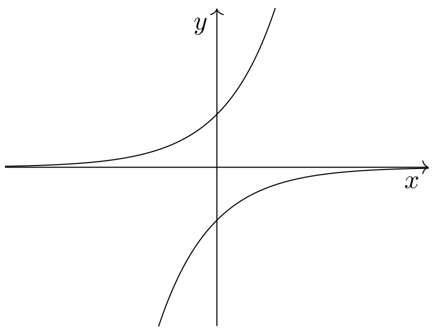
1710. This is a quartic in 2^x. Setting $z=2^x$, we have

$$\begin{aligned}&z^4-z^3+z^2-z=0\\ \Longrightarrow\ & z(z^3-z^2+z-1)=0\\ \Longrightarrow\ & z(z-1)(z^2+1)=0.\end{aligned}$$

The quadratic factor is irreducible, so $2^x=0,1$. The former has no roots, so the solution is $x=0$.

1711. The rows and columns are, as far as the rook is concerned, symmetrical. So, the first rook can be placed anywhere, without loss of generality. The first rook threatens 14 squares. Hence, with 63 squares remaining, the probability that the second rook is placed on one of the threatened squares is $\frac{14}{63}=\frac{2}{9}$.

1712. We can write $y=-\frac{1}{2^x}$ as $y=-2^{-x}$. Hence, the transformation may be seen as two reflections, one in the x axis and one in the y axis. These two reflections, switching (x,y) for $(-x,-y)$, combine to give rotation by 180° around the origin:

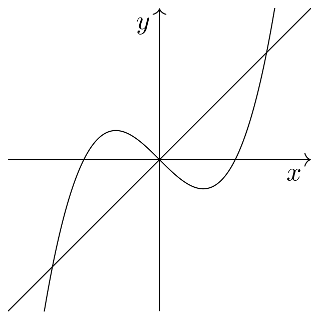


1713. Let x,y,z be $n, n+1, n+2$. Expanding binomially,

$$\begin{aligned}&n^3+(n+1)^3+(n+2)^3\\ &\equiv n^3+n^3+3n^2+3n+1+n^3+6n^2+12n+8\\ &\equiv 3n^3+9n^2+15n+9.\end{aligned}$$

This is obviously divisible by 3. Also, substituting $n=-1$ gives $-3+9-15+9=0$. Hence, by the factor theorem, the central integer $(n+1)$ is also a factor. QED.

1714. Solving simultaneously, $x=x^3-x$, which gives $x=0,\pm\sqrt{2}$. The gradient of the curve is given by $\frac{dy}{dx}=3x^2-1$. Testing this at our x values, we have $m_{\text{tangent}}=-1$ at $x=0$. Hence, $y=x$ is normal to the curve at the origin.



—— Alternative Method ——

For small x, x^3 is negligible compared to x. So, at the origin, the curve is approximately $y=-x$. This is normal to $y=x$.

1715. The area is given by $A=xy$. Differentiating both sides with respect to t,

$$\frac{dA}{dt}=\frac{dx}{dt}y+x\frac{dy}{dt}.$$

Substituting values,

$$\frac{dA}{dt}=-2\times 6+4\times 3=0.$$

1716. We need a,b,c to satisfy

$$\begin{aligned}-3&=a+b+c\\ 1&=9a-3b+c\\ 6&=4a+2b+c.\end{aligned}$$

We can easily eliminate c. The first two equations give $4=8a-4b$; the second two give $-5=5a-5b$. Hence, $a=2$, $b=3$. Substituting back in, $c=-8$. So, the required parabola is $y=2x^2+3x-8$.

1717. Since cubing preserves sign, the implication goes both ways: $\sqrt[3]{xy}=z \iff xy=z^3$.

1718. Multiplying out and differentiating,

$$\begin{aligned}&\mathrm{f}(x)=8e^x-16e^{2x}\\ \Longrightarrow\ &\mathrm{f}'(x)=8e^x-32e^{2x}.\end{aligned}$$

Setting $\mathrm{f}'(x)=0$, $\mathrm{f}(x)$ is stationary when

$$\begin{aligned}&8e^x-32e^{2x}=0\\ \Longrightarrow\ &8e^x(1-4e^x)=0.\end{aligned}$$

Since e^x is always positive, $e^x={}^1\!/_4$. We substitute this into $\mathrm{f}(x)$ directly, giving $8\cdot{}^1\!/_4(1-2\cdot{}^1\!/_4)=1$, as required.

1719. Projecting from ground level, the time of flight is given by $0=10\sin(30°)t-\frac{1}{2}gt^2$, so $t=1.02...$ seconds.

From 1 m above ground level, time of flight is given by $-1=10\sin(30°)t-\frac{1}{2}gt^2$, so $t=1.19...$ seconds. The extra horizontal range is given by $10\cos 30°\times(1.19-1.02)=1.48...\approx 1.5$ m.

1720. Four, five, six is more probable. Out of $6^3 = 216$, ${}^3\mathrm{C}_1 = 3$ outcomes yield $\{5, 5, 6\}$, while $3! = 6$, i.e. twice as many outcomes yield $\{4, 5, 6\}$.

1721. (a) This doesn't have to be true. There could be many pupils who scored exactly the median mark. In that case, more than half the scores will be greater than or equal to the median.

(b) There is no particular reason why this should be true. The mode could be anywhere.

(c) This must be true. The median is 55. Consider the five marks $40, 55, ..., 100$ as four classes. The lower half of the data is all less than 15 away from 55. The highest quarter of the data is all more than 30 away from 55, which is enough on its own to raise the mean above the median 55.

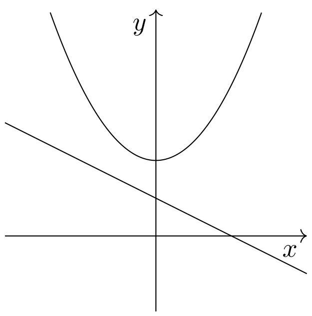
1722. (a) The cubic has a stationary point of inflection, so, since it is monic, it must be a translation of the basic cubic $y = x^3$: all other cubics have either no stationary points or else two which are not points of inflection. The point $x = 2$ is the image of O under this translation, which is by vector $\binom{2}{c}$ for some constant c. This gives $\mathrm{g}(x) = (x-2)^3 + c$.

(b) We can now use the information that $x = 2$ is a fixed point. This says that $\mathrm{g}(2) = 2$, which means $c = 2$.

1723. The first factor yields no roots, as the range of the cosine function is $[-1, 1]$. The second factor yields four:

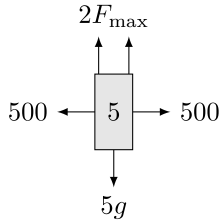
$$\begin{aligned} & 2\cos 2x - 1 = 0 \\ \Longrightarrow\ & \cos 2x = \tfrac{1}{2} \\ \Longrightarrow\ & 2x = \tfrac{\pi}{3}, \tfrac{5\pi}{3}, \tfrac{7\pi}{3}, \tfrac{11\pi}{3} \\ \Longrightarrow\ & x = \tfrac{\pi}{6}, \tfrac{5\pi}{6}, \tfrac{7\pi}{6}, \tfrac{11\pi}{6}. \end{aligned}$$

1724. Let $(p, q) = (x, y)$. Then we are restricted to points on the parabola $y = x^2 + 1$ below:



The line is $x + 2y = 1$. Solving for intersections, we have $\frac{1}{2}(1 - x) = x^2 + 1$, which has discriminant $\Delta = -\frac{7}{4} < 0$. Hence, the parabola is always above the line. So, since we are restricted to points on the parabola, we know that $x + 2y > 1$. Using the original variables, $p + 2q > 1$, as required.

1725. The boundary case is when the wood is on the point of slipping, despite maximal horizontal force. In this case, the force diagram is as follows:



Using $F_{\max} = \mu R$, we get $2\mu \times 500 - 5g = 0$. Therefore, we require $\mu \geq \frac{5g}{1000} = 0.049$.

1726. Assume, for a contradiction, that every exterior angle of a hexagon satisfies $\beta > \frac{\pi}{3}$ radians. Then the sum of the exterior angles must satisfy

$$S > 6 \times \tfrac{\pi}{3} = 2\pi.$$

But the exterior angles sum to $S = 2\pi$. This is a contradiction. So, a hexagon must have at least one exterior angle β satisfying $\beta \leq \frac{\pi}{3}$ radians. $\square$

1727. Firstly, since $u_1 = 1$ and $a, b > 0$, we know that the sequence is increasing: each term exceeds the last. The differences are given by

$$\begin{aligned} u_{n+1} - u_n &= au_n + b - u_n \\ &\equiv (a-1)u_n + b. \end{aligned}$$

Since u_n increases, so does this difference, so the sequence cannot be arithmetic. The ratios, then, are given by

$$\begin{aligned} \frac{u_{n+1}}{u_n} &= \frac{au_n + b}{u_n} \\ &\equiv a + \frac{b}{u_n}. \end{aligned}$$

Since u_n increases without bound, this ratio must decrease, tending to a. Hence, the sequence tends towards a GP, but it isn't one.

1728. Setting $z = \sqrt{x}$,

$$\begin{aligned} & \frac{5z}{2z+1} - \frac{2z-1}{z} = \frac{1}{2} \\ \Longrightarrow\ & 10z^2 - 2(4z^2 - 1) = z(2z+1) \\ \Longrightarrow\ & z = 2 \end{aligned}$$

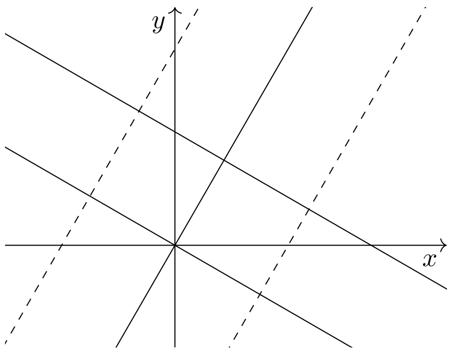
So, $\sqrt{x} = 2$, giving $x = 4$.

1729. This is incorrectly formulated because it refers to the sample, rather than the population. In order for a hypothesis test (indeed, in order for virtually all of statistics) to be meaningful, it must *use* a sample to *analyse* a population. The hypotheses must refer to probabilities in the population. In this case: "$H_0 : p = 0.5$, where p is the probability that any particular job in the population (of jobs) goes to a woman."

1730. The functions of even degree have a minimum at $0 \mapsto 1$. The function of odd degree does not.

(a) $[1, 5]$,
(b) $[-7, 9]$,
(c) $[1, 17]$.

1731. The first two lines $y = \sqrt{3}x$ and $\sqrt{3}y = -x$ are perpendicular and meet at the origin. The third is parallel to the second. The possibilities for the last side are shown as dashed lines.



The dashed lines are rotations of $\sqrt{3}x + 3y = 5$ by 90° around the origin. So, their equations are

$$\sqrt{3}y - 3x = 5,$$
$$\sqrt{3}y - 3x = -5.$$

1732. The rotation must map one vertex onto the other, so P is the midpoint of the vertices. Completing the square, the vertices are at

$$\left(\tfrac{1}{2}a, b - \tfrac{1}{4}a^2\right) \text{ and } \left(\tfrac{1}{2}c, d - \tfrac{1}{4}c^2\right).$$

Taking the mean of these, P must be at

$$\left(\frac{\frac{a}{2} + \frac{c}{2}}{2}, \frac{b - \frac{1}{4}a^2 + d - \frac{1}{4}c^2}{2}\right).$$

Multiplying top and bottom of the x coordinate by 2 and of the y coordinate by 4, the coordinates of P are

$$\left(\frac{a+c}{4}, \frac{4(b+d) - a^2 - c^2}{8}\right).$$

1733. Let F and G be polynomial functions such that $\mathrm{F}'(x) = \mathrm{f}(x)$ and $\mathrm{G}'(x) = \mathrm{g}(x)$. Then,

$$\begin{aligned} &\int_0^x \mathrm{f}(t)\,dt \equiv \int_0^x \mathrm{g}(t)\,dt \\ \Longrightarrow\ &\Big[\mathrm{F}(t)\Big]_0^x \equiv \Big[\mathrm{G}(t)\Big]_0^x \\ \Longrightarrow\ &\mathrm{F}(x) - \mathrm{F}(0) \equiv \mathrm{G}(x) - \mathrm{G}(0). \end{aligned}$$

Differentiating both sides with respect to x, the constants F(0) and G(0) go:

$$\begin{aligned} &\tfrac{d}{dx}\big(\mathrm{F}(x) - \mathrm{F}(0)\big) \equiv \tfrac{d}{dx}\big(\mathrm{G}(x) - \mathrm{G}(0)\big) \\ \Longrightarrow\ &\mathrm{f}(x) \equiv \mathrm{g}(x), \text{ as required.} \end{aligned}$$

1734. (a) A fixed point of g satisfies $\mathrm{g}(x) = x$. Rearranging this,

$$\begin{aligned} &\frac{8x^3 + 14x^2 - 1224}{553} = x \\ \Longrightarrow\ &8x^3 + 14x^2 - 1224 = 553x \\ \Longrightarrow\ &8x^3 + 14x^2 - 553x - 1224 = 0. \end{aligned}$$

(b) Running the iteration, $x_n \to -2.25$.

(c) Since $x = -9/4$ is a root, we know that $(4x+9)$ is a factor. This gives

$$\begin{aligned} &8x^3 + 14x^2 - 553x - 1224 \\ \equiv\ &(4x+9)(2x^2 - x - 136) \\ \equiv\ &(4x+9)(2x-17)(x+8). \end{aligned}$$

1735. Assuming positive x, the inequalities are

$$\begin{aligned} \log_2 x < 5 &\Longrightarrow x < 32, \\ \log_4 x > \tfrac{12}{5} &\Longrightarrow x > 27.8... \end{aligned}$$

Over the naturals, the intersection of these sets is $\{28, 29, 30, 31\}$.

1736. There are two successful outcomes RY and YR, which are equally likely. This gives

$$\begin{aligned} p &= 2 \times \frac{m}{m+n} \times \frac{n}{m+n-1} \\ &\equiv \frac{2mn}{(m+n)(m+n-1)}. \end{aligned}$$

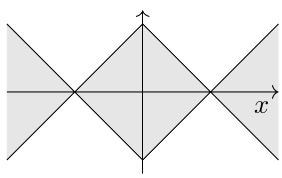
1737. Assuming the cube has unit side length, the space diagonal AB has length $\sqrt{3}$. $|AM|$ and $|MB|$ are both $\sqrt{5}/2$. Using the cosine rule in $\triangle ABC$,

$$\begin{aligned} \cos\theta &= \frac{\frac{5}{4} + \frac{5}{4} - 3}{2 \cdot \frac{5}{4}} \\ &= -\tfrac{1}{5}. \end{aligned}$$

Therefore, $\theta = 101.5°$ (1dp).

1738. (a) The boundaries of the region are given by the boundary equation $(y - |x| + 1)(y + |x| - 1) = 0$, which is satisfied when either $y - |x| + 1 = 0$ or $y + |x| - 1 = 0$. These are $\vee$-shaped modulus graphs, each consisting of two line segments. So, we have a boundary graph consisting of four line segments.

(b) The inequality is satisfied either on one or more of the boundary lines, or when exactly one factor is negative. This gives all regions vertically between the two mod graphs:



1739. The interior angles sum to $(5-2)\pi = 3\pi$ rad. Hence, by symmetry, the middle angle is $\frac{3}{5}\pi$ rad. And the smallest angle must be greater than zero. This gives upper bounds (not attained) of

(a) $\frac{6}{5}\pi$ for the largest angle,

(b) $\frac{3}{10}\pi$ for the common difference.

1740. On Earth, the time of flight satisfies

$$0 = u\sin\theta t - \tfrac{1}{2}gt^2$$
$$\therefore \quad t = \frac{2u\sin\theta}{g}.$$

Horizontal speed is $u\cos\theta$. So, the range is

$$d = \frac{2u^2\sin\theta\cos\theta}{g}.$$

As seen in this formula, the range d is inversely proportional to the gravitational acceleration g. Hence, scaling the gravitational acceleration by $\frac{1}{6}$ scales the range by 6. So, range on the Moon is around $6d$, as required.

— Nota Bene —

Air resistance would clearly affect objects on the Earth and the Moon differently. However, since the question states that these are *projectiles*, not merely objects, we are implicitly assuming that air resistance can be neglected, even on Earth.

1741. The coefficient of x^2 requires $a = 3$. So,

$$3X^2 = 12x^2 - 12x + 1.$$

The coefficient of x then requires $b = 5$. To match the constant terms, $c = -11$. This gives

$$3X^2 + 5X - 11.$$

— Alternative Method —

Rearranging to make x the subject,

$$x = \tfrac{1}{2}(X+1).$$

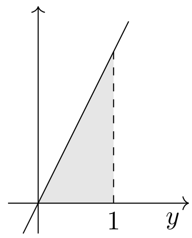
Substituting this into the original expression,

$$\begin{aligned} &12x^2 - 2x - 13 \\ &= 3(X+1)^2 - (X+1) - 13 \\ &\equiv 3X^2 + 5X - 11. \end{aligned}$$

1742. (a) Since the binomial distribution $B(10, 0.5)$ has $p = q = 0.5$, it is symmetrical about $x = 5$. Therefore, the probability of values either side of that, i.e. a and $10 - a$, must be equal.

(b) The values 2 and 8 are equally likely, as in part (a). So, the probability of either, having restricted the possibility space to $\{2, 8\}$, is $\frac{1}{2}$.

1743. Thinking graphically, the integral statement says that the triangle below has area 1:



The triangle's height must be 2, which gives the required function g as $\text{g}(y) = 2y$.

— Alternative Method —

Let $\text{g}(y) = ay + b$, for some constants a, b. Since $\text{g}(0) = 0$, we know that $b = 0$. Integrating,

$$\begin{aligned} &\int_0^1 ay\,dy = 1 \\ \Longrightarrow\ &\left[\tfrac{1}{2}ay^2\right]_0^1 = 1 \\ \Longrightarrow\ &\tfrac{1}{2}a = 1 \\ \Longrightarrow\ &a = 2. \end{aligned}$$

So, $\text{g}(y) = 2y$.

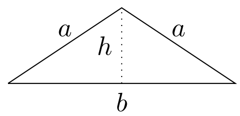
1744. Since $a = b$ is a root of the expression, $(a - b)$ is a factor. Taking it out,

$$a^3 - b^3 \equiv (a-b)\big(a^2 + ab + b^2\big).$$

— Nota Bene —

This is the factorisation of a *difference of two cubes*. Such a factorisation exists for any $a^n - b^n$, since $a = b$ is always a root of such an expression.

1745. (a) We define a and b as follows:



The perimeter gives $2a + b = 18$. Splitting the base, the height is given by $h^2 = a^2 - \frac{1}{4}b^2$, so

$$\begin{aligned} &\tfrac{1}{2}b\sqrt{a^2 - \tfrac{1}{4}b^2} = 12 \\ \Longrightarrow\ &b\sqrt{4a^2 - b^2} = 48. \end{aligned}$$

(b) The second equation squares to $4a^2 - b^2 = \frac{48^2}{b^2}$. Substituting for a gives

$$\begin{aligned} &4(9 - \tfrac{1}{2}b)^2 - b^2 = \frac{48^2}{b^2} \\ \Longrightarrow\ &324 - 36b = \frac{2304}{b^2} \\ \Longrightarrow\ &324b^2 - 36b^3 = 2304 \\ \Longrightarrow\ &b^3 - 9b^2 + 64 = 0, \quad \text{as required.} \end{aligned}$$

(c) Using a polynomial solver, the integer root is $b = 8$. So, the lengths are $(5, 5, 8)$.

— Alternative Method —

The Newton-Raphson iteration is

$$b_{n+1} = b_n - \frac{b_n^3 - 9b_n^2 + 64}{3b_n^2 - 18b_n}.$$

Running this iteration with $b_0 = 1$ or $b_0 = 5$, we get a non-integer root. Running it with $b_0 = 10$, we get $b_n \to 8$. This gives $(5, 5, 8)$ as the lengths of the triangle.

1746. Multiplying out,

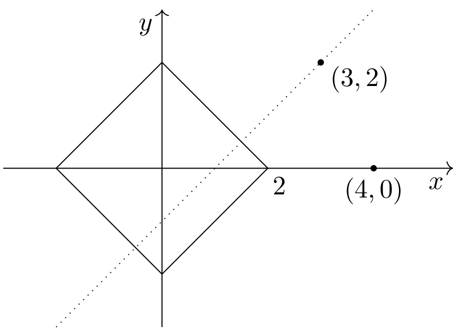
$$\begin{aligned} y &= 2 + \cos x - \cos^2 x \\ \Longrightarrow \tfrac{dy}{dx} &= -\sin x + 2\cos x \sin x \\ &\equiv \sin x(2\cos x - 1). \end{aligned}$$

— Alternative Method —

Using the product rule directly,

$$\begin{aligned} y &= (1 + \cos x)(2 - \cos x) \\ \Longrightarrow \tfrac{dy}{dx} &= -\sin x(2 - \cos x) + (1 + \cos x)\sin x \\ &\equiv \sin x(2\cos x - 1). \end{aligned}$$

1747. The square consists of $x + y = 2$ in the positive quadrant, and then similar lines in the other three quadrants. The gradient of the relevant side is 1. So, the closest distance from $(3, 2)$ lies along the perpendicular $y = x - 1$:



The distance from $(4, 0)$ to the square is clearly 2. The distance from $(3, 2)$ is along a perpendicular, the line $y = x - 1$. Solving simultaneously, the closest point is $(3/2, 1/2)$. This gives the distance as $\sqrt{4.5} > 2$. Hence, $(4, 0)$ is closer.

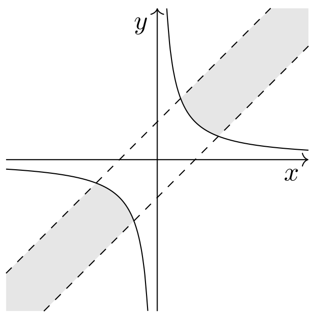
1748. Using various index laws,

(a) $3^{2x} \equiv (3^x)^2$,

(b) $3^{2x-1} \equiv \dfrac{(3^x)^2}{3}$,

(c) $3^{1-2x} \equiv \dfrac{3}{(3^x)^2}$.

1749. The boundary equations are $xy = 1$, which is a hyperbola, and $x - y = \pm 1$, which is a pair of straight lines. The region we want is between the straight lines and outside the hyperbola, in either the positive or negative quadrants:



1750. (a) Differentiating,

$$\frac{dy}{dx} = 2x + 1.$$

Hence, the gradient of the tangent at $\pm k$ is $m = \pm 2k + 1$. The first tangent has gradient $2k + 1$ and passes through $(k, k^2 + k)$. Using $y - y_1 = m(x - x_1)$, it has equation

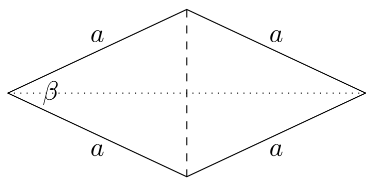
$$y - (k^2 + k) = (2k + 1)(x - k).$$

The second tangent has gradient $-2k + 1$ and passes through $(-k, k^2 - k)$. It has equation

$$y - (k^2 - k) = (-2k + 1)(x + k).$$

(b) Setting $x = 0$, both tangents cross the y axis at $y = -k^2$. This proves the result.

1751. The rhombus is



Using the cosine rule, the vertical diagonal has length given by

$$\begin{aligned} d^2 &= a^2 + a^2 - 2a^2 \cos\beta \\ \therefore \quad d &= a\sqrt{2 - 2\cos\beta}. \end{aligned}$$

The other interior angle is given, then, by $180° - \beta$. Using the identity $\cos(180° - \beta) \equiv -\cos\beta$ (easily proved on the unit circle), the horizontal diagonal has length given by

$$\begin{aligned} d^2 &= a^2 + a^2 - 2a^2 \cos(180° - \beta) \\ \therefore \quad d &= a\sqrt{2 - 2\cos(180° - \beta)} \\ &= a\sqrt{2 + 2\cos\beta}. \end{aligned}$$

So, $d_1, d_2 = a\sqrt{2 \pm 2\cos\beta}$, as required.

1752. (a) False. Variable y may depend on variable x in a non-linear way. A counterexample is $y = x^2$ and $c = 0$, which gives $z = ax + bx^2$. This is quadratic in x.

(b) True. If x and y are linearly related, then $y = px + q$ for some constants p and q. This gives $z = ax + b(px + q)$, which we can write as $z = (a + bp)x + bq$. Since $(a + bp)$ and bq are constants, this is a linear relationship.

1753. (a) That the pulleys are smooth and the strings are light.

— Nota Bene —

"Air resistance" would be an incorrect answer, since the system is in equilibrium, i.e. it isn't moving through the air. It would, however, be correct to say that we must assume that there is no wind.

(b) The system is in equilibrium, so $a = 0$ for all masses. Hence, it doesn't matter whether the strings are potentially extensible or not: we know they are not extending in this scenario.

(c) Vertically, $R = 4g$ for the block on the table, so $F_{\max} = 2g$ in limiting equilibrium. There are two possible cases: equilibrium on the point of sliding rightwards or leftwards. We deal with these case by case, resolving along the taut strings for the whole system.

① On the point of sliding leftwards:

$$\begin{aligned} mg - 2g - 2g &= 0 \\ \Longrightarrow m &= 4. \end{aligned}$$

② On the point of sliding rightwards:

$$\begin{aligned} 2g - 2g - mg &= 0 \\ \Longrightarrow m &= 0. \end{aligned}$$

Hence, since it would require $m < 0$ to make the system slide rightwards, the set of possible values of m is $[0, 4]$ kg.

1754. Carrying out the integration,

$$\begin{aligned} &\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 6x - 6\sin x \, dx \\ &= \Big[3x^2 + 6\cos x\Big]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \\ &= \left(\tfrac{12\pi^2}{9} + 6\cos\tfrac{2\pi}{3}\right) - \left(\tfrac{3\pi^2}{9} + 6\cos\tfrac{\pi}{3}\right). \end{aligned}$$

Quoting exact values, this is

$$\begin{aligned} &\left(\tfrac{12\pi^2}{9} - 3\right) - \left(\tfrac{3\pi^2}{9} + 3\right) \\ &= \pi^2 - 6. \end{aligned}$$

1755. (a) The number of handshakes is the number of ways of choosing two people to shake hands. This gives ${}^{10}\mathrm{C}_2 = 45$.

(b) With n people, there are ${}^{n}\mathrm{C}_2$ handshakes. Using the factorial formula for ${}^{n}\mathrm{C}_r$, this is $\frac{1}{2}n(n-1)$. So,

$$\begin{aligned} \tfrac{1}{2}n(n-1) &= 1485 \\ \Longrightarrow n^2 - n - 2970 &= 0 \\ \Longrightarrow n &= -54, 55 \end{aligned}$$

Therefore, $n = 55$.

1756. The shortest path is perpendicular to the line. So, we need the perpendicular to $2x + 3y + 5 = 0$ through $(3/2, 6)$. This is

$$3x - 2y + 15/2 = 0.$$

Solving simultaneously, the lines intersect at point Q, with coordinates $(-5/2, 0)$.

— Alternative Method —

The line is $y = -\frac{2}{3}x - \frac{5}{3}$. So, the *squared* distance s is given by

$$\begin{aligned} s &= \left(x + \tfrac{5}{2}\right)^2 + \left(-\tfrac{2}{3}x - \tfrac{5}{3}\right)^2 \\ &\equiv \tfrac{13}{36}(2x+5)^2. \end{aligned}$$

This is minimised at $x = -5/2$. So, the coordinates of Q are $(-5/2, 0)$.

1757. Taking natural logs of both sides,

$$\begin{aligned} & y = bx^n \\ \Longrightarrow\ & \ln y = \ln(bx^n) \\ \Longrightarrow\ & \ln y = \ln b + \ln x^n \\ \Longrightarrow\ & \ln y = \ln b + n\ln x. \end{aligned}$$

This is a linear relationship between $\ln y$ and $\ln x$, as required.

1758. The number 1 can be placed somewhere without loss of generality. Consider, then, the number 2. The probability that its placement allows for an ascending or descending sequence is $\frac{2}{5}$, as it can occupy either of the spaces next to the 1. Once these two are placed, there is only one successful location for the other numbers. This gives

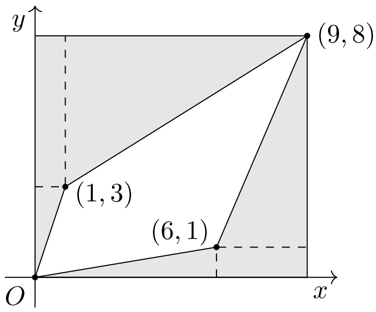
$$p = \tfrac{2}{5} \times \tfrac{1}{4} \times \tfrac{1}{3} \times \tfrac{1}{2} \times \tfrac{1}{1} = \tfrac{1}{60}.$$

— Alternative Method —

There are 6! equally likely outcomes. Of these, there are 12 successful outcomes, 2 for each of the 6 possible positions for the 1. This gives

$$p = \tfrac{12}{6!} = \tfrac{1}{60}.$$

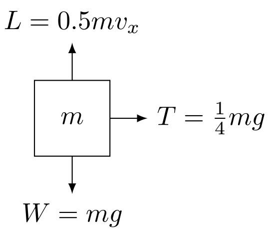
1759. We bound the quadrilateral in a rectangle whose sides are parallel to the x and y axes:



The bounding rectangle has area $8 \times 9 = 72$. We then have two unwanted rectangles with areas 5 and 3, and four unwanted triangles with areas (clockwise from left) $3/2, 20, 21/2, 3$. Hence, the area of the quadrilateral is

$$\begin{aligned} & 72 - 5 - 3 - \tfrac{3}{2} - 20 - \tfrac{21}{2} - 3 \\ = \ & 29 \text{ square units.} \end{aligned}$$

1760. (a) At the last instant for which the wheels are in contact with the ground, we model both the reaction force and the vertical acceleration as being zero. At that point, a force diagram for the glider is



Vertically, resultant force is zero (the plane is on the point of leaving the ground, but hasn't yet left it), so

$$0.5mv_x - mg = 0.$$

Solving this with $g = 10$ gives $v_x = 20 \text{ ms}^{-1}$.

(b) Horizontally, the acceleration is $\frac{1}{4}g = 2.5$ ms^{-2}, $u = 0$ and $v = 20$. So, *suvat* gives

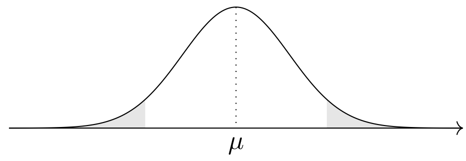
$$\begin{aligned} 20^2 &= 0^2 + 2 \cdot 2.5 s_x \\ \Longrightarrow s_x &= 80. \end{aligned}$$

The initial distance was 100 m, so the glider takes off 20 m from the winch.

1761. The sum is an infinite geometric series, with first term x and common ratio x.

$$\begin{aligned} & \sum_{r=1}^{\infty} x^r = \tfrac{1}{2} \\ \Longrightarrow \ & \frac{x}{1-x} = \tfrac{1}{2} \\ \Longrightarrow \ & x = \tfrac{1}{3}. \end{aligned}$$

1762. This is true. A normal distribution is symmetrical about μ, and a two-tail test must be symmetrical by definition.



This means that both the critical and acceptance regions must be symmetrical about μ. This isn't true for other distributions, e.g. the binomial, nor for one-tailed tests.

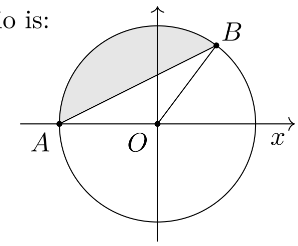
1763. (a) Since $\text{f}(x) = 0$ has roots at $x = \alpha$ and $b = \beta$, we know that $\text{f}(\alpha) = \text{f}(\beta) = 0$. Hence,

$$\begin{aligned} \int_\alpha^\beta \text{g}(x)\,dx &= \Big[\,\text{f}(x)\Big]_\alpha^\beta \\ &\equiv \text{f}(\beta) - \text{f}(\alpha) \\ &= 0. \end{aligned}$$

(b) Likewise using $\text{f}(\alpha) = \text{f}(\beta) = 0$,

$$\begin{aligned} \int_\alpha^\beta \frac{2x + \text{g}(x)}{3}\,dx &= \tfrac{1}{3}\Big[x^2 + \text{f}(x)\Big]_\alpha^\beta \\ &\equiv \tfrac{1}{3}\big(\beta^2 + \text{f}(\beta) - \alpha^2 - \text{f}(\alpha)\big) \\ &= \tfrac{1}{3}\big(\beta^2 - \alpha^2\big). \end{aligned}$$

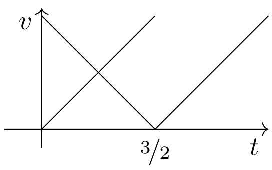
1764. The scenario is:



Triangle AOB has lengths $\big(1, 1, \sqrt{3.2}\big)$. Using the cosine rule, angle AOB is 2.214 radians. The area of $\triangle AOB$ is $\frac{1}{2}\sin 2.214 = 0.4$. The area of sector AOB is $\frac{1}{2} \cdot 1^2 \cdot 2.214 = 1.107$. So, the area of the shaded segment is $1.107 - 0.4 = 0.707 \approx 0.71$.

1765. (a) Differentiating gives the velocity as $v_1 = 2t - 3$. The speed is the magnitude of this quantity, which is encoded algebraically as $v_1 = |2t - 3|$. The speed of the second particle is given, in a similar fashion, by $v_2 = |-2t|$.

(b) Sketching the speed-time graphs:



By symmetry, the point of intersection is halfway to $\frac{3}{2}$, at $t = \frac{3}{4}$. Hence, the first particle is moving faster for $t \in \big[0, \frac{3}{4}\big)$.

1766. The curve is $y = k(x^4 - x^2 + 1)$. For SPs, we set the first derivative to zero:

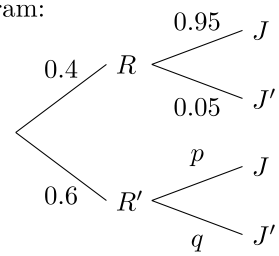
$$\begin{aligned} & k(4x^3 - 2x) = 0 \\ \implies & x(2x^2 - 1) = 0 \\ \implies & x = 0, \pm\tfrac{1}{\sqrt{2}}. \end{aligned}$$

Substituting these back in, there are SPs at $(0, k)$ and $(\pm 1/\sqrt{2}, 3k/4)$. The latter pair have the same y coordinate.

— Nota Bene —

This result is due to the even symmetry of the graph (no odd powers), and would be true of any curve $y = ax^4 + bx^2 + c$ with a stationary point at $x \neq 0$.

1767. (a) Tree diagram:



(b) Since $\mathbb{P}(J) = 0.83$, we have

$$\begin{aligned} & 0.83 = 0.4 \times 0.95 + 0.6p \\ \implies & p = 0.75. \end{aligned}$$

(c) For $\mathbb{P}(R \mid J)$, we restrict the possibility space to the first and third branches, and calculate the probability of the first:

$$\begin{aligned} \mathbb{P}(R \mid J) &= \frac{0.4 \times 0.95}{0.4 \times 0.95 + 0.6 \times 0.75} \\ &= \tfrac{38}{83}. \end{aligned}$$

1768. Assume, for a contradiction, that (a, a, b) is a Pythagorean triple. The hypotenuse must be longer than the other two sides, so $a^2 + a^2 = b^2$, which gives

$$2a^2 = b^2.$$

The RHS is a square, so must have an even number of factors of 2. By the same logic, the LHS must have an odd number of factors of 2, since it has one more than a square. This is a contradiction. Hence, the triangle is not isosceles. □

1769. Solving to find intersections,

$$\begin{aligned} & x^4 + x^2 = x^3 + x \\ \implies & x^4 - x^3 + x^2 - x = 0 \\ \implies & x(x^3 - x^2 + x - 1) = 0 \\ \implies & x(x - 1)(x^2 + 1) = 0. \end{aligned}$$

The quadratic factor has no real roots. Hence, the intersections are $x = 0, 1$. To set up an integral, we note that the cubic is above the quartic for $x \in (0, 1)$. So, the area of the shaded region is

$$\begin{aligned} & \int_0^1 x^3 + x - x^4 - x^2 \, dx \\ = & \left[\tfrac{1}{4}x^4 + \tfrac{1}{2}x^2 - \tfrac{1}{5}x^5 - \tfrac{1}{3}x^3\right]_0^1 \\ = & \left(\tfrac{1}{4} + \tfrac{1}{2} - \tfrac{1}{5} - \tfrac{1}{3}\right) - (0) \\ = & \tfrac{13}{60}, \text{ as required.} \end{aligned}$$

1770. Parts (a) and (b) are the components of the total force in (c):

(a) $mg \sin\theta$ N,

(b) $mg \cos\theta$ N,

(c) mg N.

1771. Since there are infinitely many points satisfying the equations, the two lines must be the same. The scale factor is 2, giving $a = -1$, $b = 6$.

1772. We can see this as two successive transformations: reflection in $y = x$, which switches the coordinates to $x^2 + 3y^2 = 1$, followed by translation by vector $\binom{-2}{2}$. Under these transformations consider e.g. the point $(0, 1)$. This transforms to $(1, 0)$, then to $(-1, 2)$. So, the mirror line is the perpendicular bisector of $(0, 1)$ and $(-1, 2)$, which is $y = x + 2$.

1773. The boundary equation $\text{f}(x) - \text{g}(x) = 0$ must be a quadratic with roots at $x = -2/3$ and $x = 6$. Hence, $\text{f}(x) - \text{g}(x) = 0$ may be written as

$$\begin{aligned} & (3x + 2)(x - 6) = 0 \\ \implies & 3x^2 - 16x - 12 = 0. \end{aligned}$$

We can also, by subtracting the given functions, write this as $3x^2 - 16x + a - b = 0$. Hence, $a - b = -12$, giving $b - a = 12$.

1774. (a) Using a calculator, $r = 0.8154... \approx 0.82$.

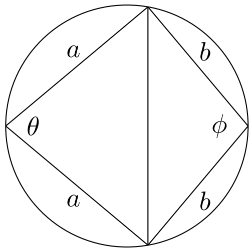
(b) Let ρ be the correlation coefficient between the variables w and c in the population of years. The hypotheses are

$$\begin{aligned} & \text{H}_0 : \rho = 0, \\ & \text{H}_1 : \rho \neq 0. \end{aligned}$$

(c) This is a two-tail test at the 5% level. The critical value is 0.8783. Since $0.8154 < 0.8783$, there is insufficient evidence to reject H_0. The researcher has not found sufficient evidence for correlation between extreme weather and civil unrest.

(d) This is certainly not a reasonable application of such a test. By their one-off nature, it is certain that neither extreme weather events nor civil unrest follow normal distributions. Hence, the assumption that the underlying population in this test is bivariate normal is not appropriate. The result of such a test will not be useful to the researcher.

1775. We can split the kite into two triangles as follows:



Using the sine area formula on the two triangles, the area of the kite is given by

$$A = \tfrac{1}{2}a^2 \sin\theta + \tfrac{1}{2}b^2 \sin\phi.$$

And, since the kite is cyclic, $\phi = 180° - \theta$. This gives $\sin\phi = \sin\theta$. Therefore, the area of K is

$$\begin{aligned} A &= \tfrac{1}{2}a^2 \sin\theta + \tfrac{1}{2}b^2 \sin\theta \\ &\equiv \tfrac{1}{2}(a^2 + b^2)\sin\theta, \text{ as required.} \end{aligned}$$

1776. The direction vector of the line is $\binom{3}{6}$. Hence, the gradient is $\frac{6}{3} = 2$. When $t = 0$, the line passes through $(1, 5)$. Hence, the equation of the line is $y - 5 = 2(x - 1)$, which simplifies to $y = 2x + 3$.

1777. Differentiating, $\frac{dy}{dx} = 6x - 5$, so the gradients are $\pm 6a - 5$. The relevant points are $(\pm a, 3a^2 \mp 5a + 2)$. Substituting these into $y = m_\pm x + c_\pm$,

$$\begin{aligned} & 3a^2 \mp 5a + 2 = (\pm 6a - 5)(\pm a) + c_\pm \\ \Longrightarrow\ & c_\pm = 3a^2 \mp 5a + 2 - (\pm 6a - 5)(\pm a) \\ \Longrightarrow\ & c_\pm = 3a^2 \mp 5a + 2 - 6a^2 \pm 5a \\ \Longrightarrow\ & c_\pm = -3a^2 + 2. \end{aligned}$$

Since the y intercept is the same for both tangent lines, they meet on the y axis, as required.

1778. Multiplying top and bottom by x^2,

$$\begin{aligned} & \frac{x^2 + x}{x^2 - 1} = x \\ \Longrightarrow\ & \frac{x(x+1)}{(x+1)(x-1)} = x \\ \Longrightarrow\ & \frac{x}{x-1} = x \\ \Longrightarrow\ & 0 = x(x-2) \\ \Longrightarrow\ & x = 0, 2. \end{aligned}$$

The former is not a root of the original equation. We introduced it by multiplying top and bottom by x^2. The solution is $x = 2$.

1779. In the σ definition, an outlier lies more than 2 standard deviations from the mean. According to this definition, the probability that a datum is not classified as an outlier is $p = 0.9545$. Hence, the probability that a sample of 10 contains at least one outlier is

$$\begin{aligned} \mathbb{P}(\text{at least one outlier}) &= 1 - \mathbb{P}(\text{no outliers}) \\ &= 1 - 0.9545^{10} \\ &= 0.372289... \\ &\approx 37\%. \end{aligned}$$

1780. (a) Splitting the fraction up,

$$y = \frac{a}{c} + \frac{b}{cx}.$$

So $p = \frac{a}{c}$, $q = \frac{b}{c}$.

(b) The transformations are a stretch, scale factor $\frac{b}{c}$, in the y direction, followed by a translation by vector $\frac{a}{c}\mathbf{j}$.

(c) The untransformed graph $y = x$ has the axes $x = 0$ and $y = 0$ as its asymptotes. The axis $x = 0$ is unaffected by the transformations, while $y = 0$ is transformed to $y = \frac{a}{c}$.

1781. This is a quadratic in 3^{-x}:

$$\begin{aligned} & 3^{-2x} + 3^{-x} - 30 = 0 \\ \Longrightarrow\ & (3^{-x} - 5)(3^{-x} + 6) = 0 \\ \Longrightarrow\ & 3^{-x} = 5, -6. \end{aligned}$$

The latter has no roots, because 3^{-x} is positive. So, $3^{-x} = 5$. This gives $x = -\log_3 5$, which can be rewritten as $x = \log_5 3$.

1782. Starting with the LHS,

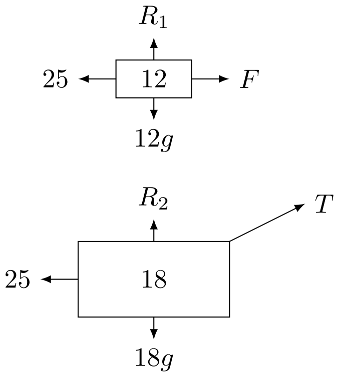
$$\begin{aligned} & \frac{x^2 + 3x + 2}{3x^3 + 4x^2 + x} \\ \equiv\ & \frac{(x+1)(x+2)}{x(3x+1)(x+1)} \\ \equiv\ & \frac{x+2}{x(3x+1)}, \quad \text{assuming } x \neq -1. \end{aligned}$$

Putting the RHS over a common denominator,

$$\begin{aligned} & \frac{2}{x} - \frac{5}{3x+1} \\ \equiv\ & \frac{2(3x+1) - 5x}{x(3x+1)} \\ \equiv\ & \frac{x+2}{x(3x+1)}. \end{aligned}$$

This proves the identity for all real numbers except $x = 0$ and $x = -\frac{1}{3}$, where both sides are undefined, and $x = -1$, where the LHS is undefined.

1783. (a) The force diagrams are:



(b) Resolving horizontally for the combined sledge and load, $T\cos 30° - 25 = 0$, giving $T = 28.9$ N (3sf).

(c) Resolving vertically for the load, $R_1 = 12g$. When the load is just about to slip, friction is at $F_{\text{max}} = \mu R_1$. This gives $\mu \times 12g - 25 = 0$, so $\mu_{\text{min}} = 0.213$ (3sf).

1784. To 2dp, these bounds of this interval are 3.14 and 3.17. As fractions, these are $\frac{314}{100}$ and $\frac{317}{100}$. Hence, the rationals $\frac{315}{100}$ and $\frac{316}{100}$ fulfil our requirements.

1785. (a) Yes. This combination gives you the common ratio, and thence the sum.

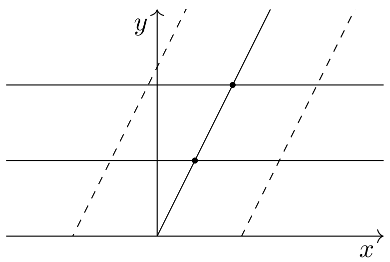
(b) No. While this combination would give you to sum for an AP, for a GP it doesn't. The common ratio isn't fixed by this information.

(c) Yes. Any three pieces of information, so long as they are not redundant, is sufficient to find all of the terms, and therefore the sum, of a geometric series.

1786. Differentiating implicitly by the chain rule,

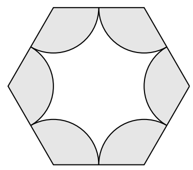
$$\begin{aligned}&\tfrac{d}{dt}(\sqrt{x} + \sqrt[3]{y})\\ &= \tfrac{d}{dt}\left(x^{\frac{1}{2}} + y^{\frac{1}{3}}\right)\\ &= \tfrac{1}{2}x^{-\frac{1}{2}}\tfrac{dx}{dt} + \tfrac{1}{3}y^{-\frac{2}{3}}\tfrac{dy}{dt}.\end{aligned}$$

1787. The two possible positions for the fourth line are the dashed lines below:



The distance along $y = 2x$ between the marked points at $y = 1, 2$ is $d = \sqrt{0.5^2 + 1^2} = \sqrt{5}/2$. So, we must translate $y = 2x$ by vector $\pm\sqrt{5}/2\,\mathbf{i}$. This gives $y = 2(x \mp \sqrt{5}/2)$. Multiplying out, the two lines are $y = 2x + \sqrt{5}$ and $y = 2x - \sqrt{5}$.

1788. The hexagon has area $6\sqrt{3}$. We need to find the shaded area:



Three of the shaded sectors sums to one circle. The radius is 1, so this gives a total of 2π shaded. Hence, the proportion shaded is

$$p = \frac{2\pi}{6\sqrt{3}} = \frac{\pi}{3\sqrt{3}}.$$

1789. Since this is a continuous distribution, the answer to (b) is zero automatically. Hence, the answers to (a) and (c) are the same. Using a calculator, to 3sf,

(a) $\mathbb{P}\big(X \in (1,2)\big) = 0.150$,

(b) $\mathbb{P}\big(X \in \{1,2\}\big) = 0$,

(c) $\mathbb{P}\big(X \in [1,2]\big) = 0.150$.

1790. Defining x to be the side opposite $\arccos\frac{7}{8}$, we can use the sine rule to calculate the other two sides. The side opposite $\arccos {}^{11}/_{16}$ is given by

$$y = \frac{\sin(\arccos {}^{11}/_{16})}{\sin(\arccos {}^{7}/_{8})}x \equiv \tfrac{3}{2}x.$$

And, using the fact that the angles in a triangle add up to 180°,

$$z = \frac{\sin(180° - \arccos {}^{7}/_{8} - \arccos {}^{11}/_{16})}{\sin(\arccos {}^{7}/_{8})}x \equiv 2x.$$

Using the perimeter, $x + \frac{3}{2}x + 2x = 9$, giving the side lengths as $(2, 3, 4)$.

1791. (a) Since a and b have unit length, $p^2 + q^2 = 1$.

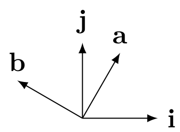
(b) Solving for $\mathbf{j}$ by elimination,

$$\begin{aligned}q\mathbf{a} &= pq\mathbf{i} + q^2\mathbf{j},\\ p\mathbf{b} &= pq\mathbf{i} - p^2\mathbf{j}.\end{aligned}$$

This gives $q\mathbf{a} - p\mathbf{b} + (q^2 + p^2)\mathbf{j}$. Hence, using part (a), $\mathbf{j} = p\mathbf{b} - q\mathbf{a}$. Substituting back in gives $\mathbf{i} = p\mathbf{a} + q\mathbf{b}$.

(c) The gradients of vectors $\mathbf{a}$ and $\mathbf{b}$ are $\frac{q}{p}$ and $-\frac{p}{q}$, which are negative reciprocals.

(d) This is a rotation by angle 60°:



1792. Numerator and denominator are zero at $x = \frac{1}{3}$, so we must cancel factors of $(3x-1)$:

$$\begin{aligned}&\lim_{x\to\frac{1}{3}} \frac{6x^2+13x-5}{12x^2-7x+1}\\ =\ &\lim_{x\to\frac{1}{3}} \frac{(3x-1)(2x+5)}{(3x-1)(4x-1)}\\ =\ &\lim_{x\to\frac{1}{3}} \frac{2x+5}{4x-1}.\end{aligned}$$

We can now take the limit, giving

$$\begin{aligned}&\frac{2\cdot\frac{1}{3}+5}{4\cdot\frac{1}{3}-1}\\ =\ &17.\end{aligned}$$

1793. Since the population is large, the probability that any particular datum sampled from it lies between the quartiles is 50%. So, $X \sim B(5, 0.5)$. Hence, the required probability is $0.5^5 = \frac{1}{32}$.

1794. Factorising the LHS,

$$\frac{d^2-a^2}{c^2-b^2} \equiv \frac{(d+a)(d-a)}{(c+b)(c-b)}.$$

Since an AP is symmetrically distributed about its mean, $(d+a) = (c+b)$. Furthermore, $(d-a)$ is three common differences, while $(c-b)$ is one; therefore $(d-a) = 3(c-b)$. Cancelling the factors yields the required result.

1795. (a) False: $a=-1$, $b=-2$ is a counterexample.
(b) True: an inequality is preserved under linear transformations $mx+c$ with $m>0$.
(c) False: $a=3$, $b=2$ is a counterexample.

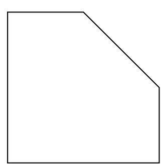
1796. The quadratic formula gives

$$x = \frac{6 \pm \sqrt{36-4k}}{2} = 3 \pm \sqrt{9-k}.$$

Hence, the distance between the x intercepts is $2\sqrt{9-k}$. Equating this to 2,

$$\begin{aligned}&2\sqrt{9-k} = 2\\ \implies\ &9-k=1\\ \implies\ &k=8.\end{aligned}$$

1797. Consider a square with one corner cut off:



This is a pentagon with two pairs of parallel sides, so is a counterexample to the claim.

1798. (a) False. These are discrete sums, so the value u_b contributes twice to the RHS, but only once on the LHS.
(b) True. Since integrals are continuous in x, the overlapping of the limits at $x=b$ contributes nothing to the integral.

1799. The possibility space is an $n\times 6$ rectangle. Of these $6n$ outcomes, there are 15 in which the six-sided die has a higher score, forming a triangle. Hence, the probability is $\frac{15}{6n} = \frac{5}{2n}$.

1800. The common difference is $d=2$, so the number of terms in the sequence is $n = \frac{2p-6}{2}+1 = p-2$. Therefore, the sum is

$$S_n = \tfrac{1}{2}(p-2)(12+2(p-3)) = (p-2)(p+3).$$

——— End of 18th Hundred ———